

## Intro to Polar Coordinates and the Unit Circle

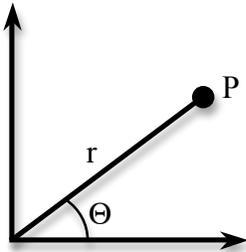
In this lesson, students construct the unit circle given basic information about polar coordinates.

Before this lesson, students should know:

- Pythagorean Theorem
- Trigonometric properties ( $\sin = \text{opp}/\text{hyp}$ ,  $\cos = \text{adj}/\text{hyp}$ ,  $\tan = \text{opp}/\text{adj}$ )
- Basic angles ( $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ ,  $360^\circ$ )

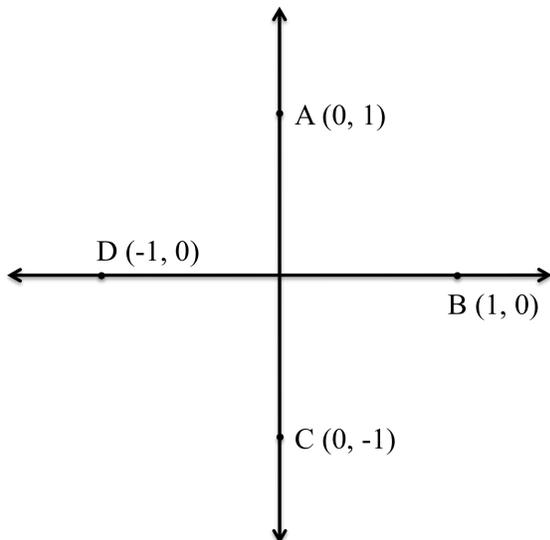
I.

Instead of  $(x, y)$ , let's denote a point by (distance from origin, angle from positive x-axis).

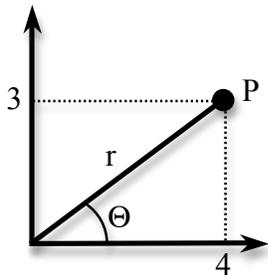


Point  $P = (r, \Theta)$ .

Write the polar coordinates for points A, B, C, and D.



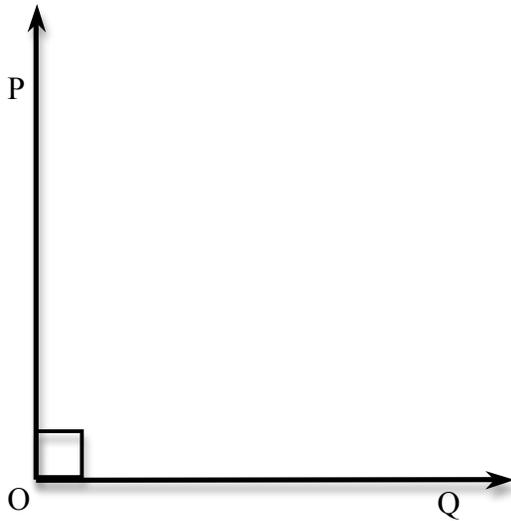
Solve for  $r$ .



Solve for  $\theta$  using trigonometric properties.

Write the polar coordinates for Point P.

II.



What is the measure of  $\angle POQ$ ?

Draw the angle bisector of  $\angle POQ$  using a dotted line. What is the angle between this line and OQ?

Split  $\angle POQ$  into thirds by drawing dotted lines. What is the measure of each angle formed between line OQ?

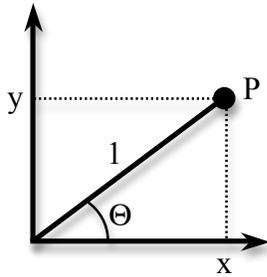
Draw an arc from P to Q, so that the area between O, P, and Q creates one-fourth a circle. Label the intersections of each dotted line and the quarter-circle A, B, C.

If Q has rectangular coordinates (1, 0), what are the polar coordinates of A, B, and C?

### III.

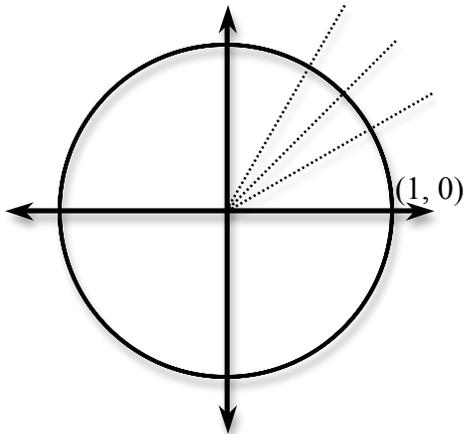
Find the rectangular coordinates of A, B, and C using trigonometric properties. Which trigonometric property should you use to calculate the x-values? The y-values? Why?

For  $r = 1$  and an angle  $\Theta$ , write the x-value and y-value for any point P.



### IV.

Extend what you did in Part II to the entire coordinate plane. (The dotted lines in the first quadrant are already drawn for you.)



Complete the same dotted lines in quadrants 2, 3, and 4. Label the degrees between each dotted line and the positive x-axis.

This is called the **Unit Circle** because the radius is 1. Based on this circle, complete the following.

$$\sin(30^\circ) = \sin(\underline{\quad})$$

$$\sin(120^\circ) = \cos(\underline{\quad})$$

$$\tan(315^\circ) = \tan(\underline{\quad})$$

$$\cos(135^\circ) = \cos(\underline{\quad})$$

$$\cos(270^\circ) = \sin(\underline{\quad})$$

$$\tan(210^\circ) = -\tan(\underline{\quad})$$