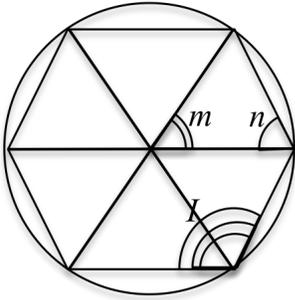## Internal Angles of Regular Polygons

Objectives:
Given that a circle is 360°, students can find the internal angles of regular polygons by splitting the polygon into triangles with vertices at the center.

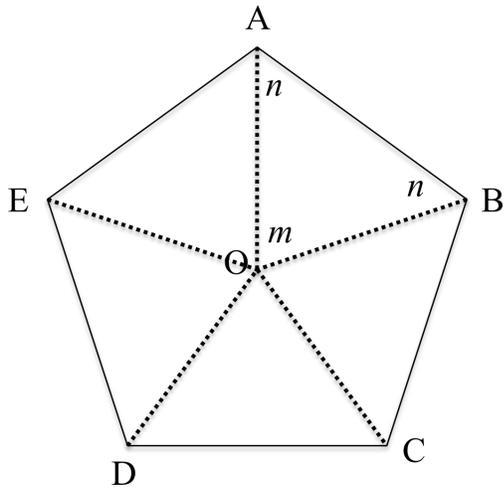In the hexagon below, *m* is the central angle; *n* is the peripheral angle; and *I* is the internal angle of the polygon.

Find the measure of ∠m and ∠n.
∠m = 60°
∠n = 60°

Will the measures of ∠m and ∠n always be the same for any regular polygon with *s* sides? Why or why not?
No, because as *s* increases, ∠m decreases and ∠n increases.

Find the measure of ∠m and ∠n.
∠m = 72°
∠n = 54°

What kind of triangle is ΔOAB, one of the internal triangles with its vertex at the center (O)?
Isosceles

Is this true for all internal triangles for any regular polygon with *s* sides? Why or why not?
Yes, because there are two angles *n* for each internal triangle.

What is the measure of ∠ABC, one of the internal angles?
∠ABC = 2n = 108°

What is the sum of the internal angles of a pentagon?
108° • 5 = 540°

You have a polygon of *s* sides. Write an equation for the measurement of the central angle, *m*.
$m = 360/s$

What is m•s?
360

Write an equation for the measurement of the peripheral angle, *n*, in terms of *m* and *s*.
$180 = m + 2n$
$n = (180 - m)/2$
$n = (180 - (360/s))/2 = 90 - 180/s$

Write an equation for the measure of each internal angle of a regular polygon.
$I = 2n = 180 - 360/s$

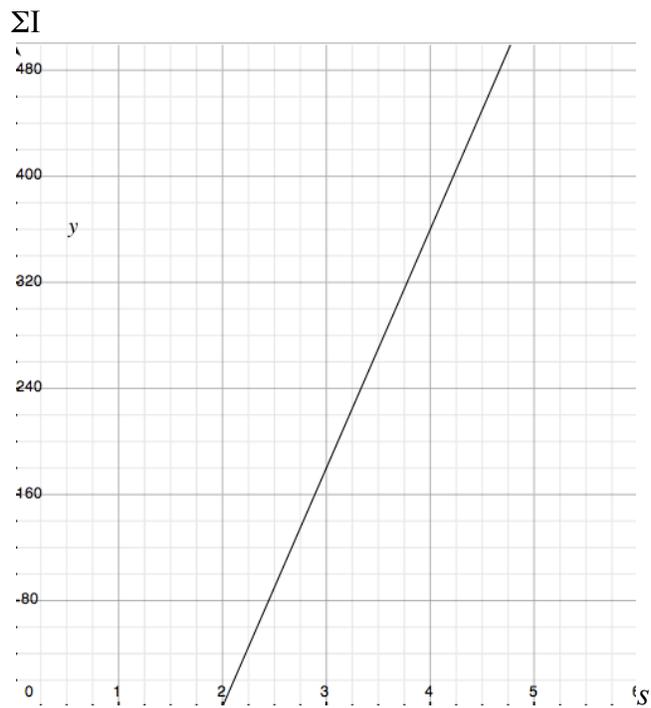Graph this equation with *s* on the x-axis and *I* on the y-axis.

I
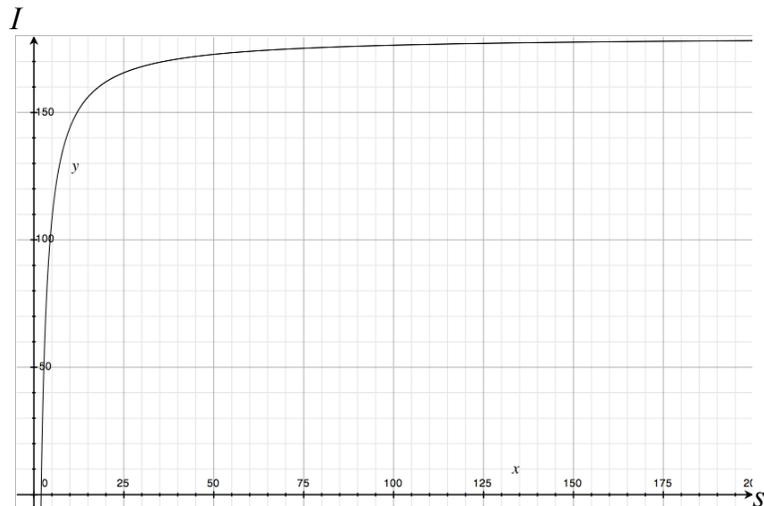
Why does it have this shape? Will *I* ever cross 180?

Write an equation for the sum of all internal angles, *I*, in terms of *s*.
ΣI = s•I = s(180 – 360/s) = 180s – 360

Graph this equation with *s* on the x-axis and ΣI on the y-axis.

ΣI

S

ΣI

What kind of function is it?
Linear

Explain conceptually what this graph means. For what values of s do these functions make sense?
This graph shows the sum of internal angles for any polygon with s sides. It only makes sense for integer values of s greater than 2.

Find the measurements of $\angle m$, $\angle n$, $\angle I$, ΣI for a polygon of 53 sides. What is m•s?
$\angle m = 6.79°$
$\angle n = 86.60°$
$\angle I = 173.21°$
$\Sigma I = 9180°$
m•s = 360

Complete the function table.

s	Measure of internal angles (I)	Sum of internal angles (ΣI)
3		
4		
5		
6		
7		